Finding Minimum Locating Arrays Using a SAT Solver

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Introduction

- Locating arrays are covering arrays of a special type.
  - Not only exercise all t-wise interactions, but also locate any t-wise faulty interactions.
  - A (d, t)-locating array can locate d t-wise faulty interactions.
- We discuss the question: how small can locating arrays be?
- We limit our discussion to (1, 2)-locating arrays.
  - Pair (2)-wise testing is the most common form of combinatorial interaction testing.

- A covering array A is (1, 2)-Locating if

$$\rho_A(T_1) \neq \rho_A(T_2) \Leftrightarrow T_1 \neq T_2 \text{ for any } T_1,\ T_2$$

  - T_1, T_2: Pair-wise interactions
  - $\rho_A(T)$: The set of rows (tests) of A that cover T
- The rows that cover T_1 must be different from the rows that cover T_2.

→ A faulty interaction can be identified by seeing failed rows (tests).

(1, 2)-locating array of size 11 x 11

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1
0	0	1	1	1	0	0	0	1	1	1
0	1	0	1	1	0	1	1	0	0	1
0	1	1	0	1	1	0	1	0	1	0
0	1	1	1	0	1	1	0	1	0	0
1	0	0	1	1	1	1	0	0	1	0
1	0	1	1	0	1	0	1	0	0	1
1	1	0	0	1	1	0	0	1	0	1
1	1	0	1	0	0	0	1	1	1	0
1	1	1	0	0	0	1	0	0	1	1

Method

The problem of finding a locating array of a given size (# rows)

↓ Formulate

Constraint satisfaction problem (CSP)

↓ Transform

Boolean satisfiability problem (SAT)

↓ Solve

Exist (SAT) / Not Exist (UNSAT)

Formulation as a CSP

Array (#values v = 2)

0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

m (rows), n (columns)

Channeling constraint: $y_{r,(i_1,i_2)} = l_1 * v + l_2$

Convert

Alternative Matrix $y_{r,(i_1,i_2)}$

(1, 2)	(1, 3)	(1, 4)	(1, 5)	(2, 3)	(2, 4)	(2, 5)…
0	0	0	0	0	0	0
0	1	1	1	1	1	1
1	0	1	1	2	3	3
1	1	0	1	3	2	3
2	2	3	2	0	1	0
2	3	2	2	1	0	0
3	2	2	2	2	2	2
3	3	3	3	3	3	3

Covering constraint:
Each value in {0, 1, 2, 3} must occur at least once in every column.

Locating constraint:

$$\forall L_1, L_2, 0 \leq L_1, L_2 \leq v^2\text{-}1,$$
$$\forall i_1, i_2, j_1, j_2, 1 \leq i_1, i_2 \leq n, 1 \leq j_1, j_2 \leq n,$$
$$\bigvee_{1 \leq r \leq m} \{(y_{r,(i_1,i_2)} = L_1)\ XOR\ (y_{r,(j_1,j_2)} = L_2)\}$$

Arrays obtained

	columns	rows	≥*	time	CAN**
#values v=2	15	15	12	1835.6	7
	16	16	12	6710.8	7
	17	16	12	10543.8	7
	18	17	12	2269.2	7
	19	18	12	1106.3	7
	20	19	12	1403.3	7
	21	21	12	1173	7
	22	21	12	3264.6	7
	23	20	12	6759.7	7
	24	NA	12	NA	7
	25	NA	12	NA	7
#values v=3	10	30	22	3523.8	14
	11	31	22	1579	14
	12	33	23	9479.4	14
	13	38	24	5493.6	14
	14	NA	24	NA	14
	15	NA	24	NA	14

* Known lower bound

** CAN: Covering Array Number

Conclusion

- We proposed a method that uses SAT solving to find a (1, 2)-locating arrays of a given size.
- We reported preliminary results obtained using the method.
- Many of the locating arrays discovered using the method are the smallest known.